

SPECIALIA

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The critical rotation of gravitating degenerate cylinders

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Summary. The dependence of critical rotational parameter $\epsilon = \Omega^2/2\pi G\rho_a$ on the parameter y_0^{-2} has been found for degenerate cylinders with the help of Padé approximations technique.

Fluid and gaseous cylinders are widely used as a model in investigations of various astrophysical objects, such as the spiral arms of galaxies. In this note, we consider the problem of the equilibrium and the critical rotation of rotating gaseous cylinders consisting of degenerate matter. The problem of the critical rotation of polytropic cylinders has been previously considered by Robe¹. The equation to be solved is²:

$$\frac{1}{\eta} \frac{d}{d\eta} \left(\eta \frac{d\varphi}{d\eta} \right) + (\varphi^2 - y_0^{-2})^{3/2} - \epsilon q^3 = 0, \quad q = (1 - y_0^{-2})^{1/2},$$

$$\epsilon = \frac{\Omega^2}{2\pi G\rho_a} \quad (1)$$

Here Ω is the angular velocity of rotation, ρ_a is the matter density at the axis of the cylinder, y_0^{-2} is the parameter of the problem, $0 \leq y_0^{-2} < 1$. The boundary conditions are given by

$$\varphi(0) = 1, \quad \varphi'(0) = 0. \quad (2)$$

The radius of the cylinder η_1 is defined by

$$\varphi(\eta_1) = y_0^{-1}. \quad (3)$$

No analytical solution of equation (1) is known for any value of the parameters y_0^{-2} and ϵ . Using the Runge-Kutta method, some authors^{2,3} have solved the equation (1) for only a few values of the parameters. Therefore, in this note we describe the effective method of approximate analytical solving of equation (1) for arbitrary values of the parameters y_0^{-2} and ϵ . The method used consists of Padé approximations technique recently applied to the study of polytropic⁴ and degenerate⁵ non-rotating spheres. We start with the series expansion of $\varphi(\eta)$ near the origin

$$\varphi = \sum_{i \geq 0} a_i \eta^{2i}, \quad a_0 = 1. \quad (4)$$

The 4 coefficients that will be used in the following are

$$a_1 = -\frac{q^3}{4} f, \quad a_2 = \frac{3q^4}{64} f, \quad f = 1 - \epsilon,$$

$$a_3 = -\frac{q^5 f}{3 \cdot 2^8} [3 + 2(1 + q^2) f], \quad (5)$$

$$a_4 = \frac{q^6 f [3 + 11(1 + q^2) f + 2(-1 + 3q^2) f^2]}{2^{14}}.$$

Then we present the function $\varphi(\eta)$ as rational form

$$\varphi = \frac{1 + A\eta^2 + B\eta^4}{1 + C\eta^2 + D\eta^4}, \quad (6)$$

where the coefficients A, B, C and D are expressed in terms of a_i 's as follows

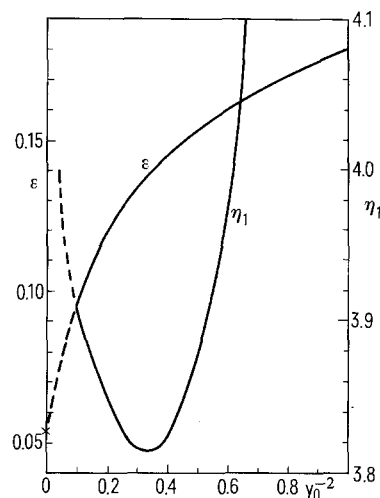
$$A = a_1 + C, \quad B = a_2 + a_1 C - D, \quad C = (a_1 a_4 - a_2 a_3)/\Delta,$$

$$D = (a_3^2 - a_2 a_4)/\Delta, \quad \Delta = a_2^2 - a_1 a_3 \quad (7)$$

Using (3, 6, 7) we find the following biquadratic for the radius η_1 of the degenerate cylinder

$$(B - D y_0^{-1}) \eta_1^4 + (A - C y_0^{-1}) \eta_1^2 + (1 - y_0^{-1}) = 0. \quad (8)$$

The critical rotation is defined by the condition that the centrifugal force balances the gravitational force at the outer boundary of cylinder. For the critical value of ϵ , it can be shown that along with the condition (3) the additional condition $\varphi'(\eta_1) = 0$ is valid.



The critical rotational parameter $\epsilon = \Omega^2/2\pi G\rho_a$ (left ordinate) and the radius η_1 of cylinder at the critical rotation (right ordinate) for the rotating degenerate cylinders as the functions of parameter y_0^{-2} . The cross marks the value of ϵ_{crit} for cylindrical polytrope $n=3$ due to Robe¹.

Results of calculations of the critical rotational parameter ϵ are presented in the figure. Also presented is the curve of the radii of cylinders at critical rotation.

The maximal (critical) value of rotational parameter ϵ_{\max} increases with increasing y_0^{-2} , that is with the transition

from relativistic cylinders to non-relativistic ones. As to the dimensionless radius η_1 of the cylinders, the function $\eta_1(y_0^{-2})$ has the minimum at $y_0^{-2} \approx 0.35$ while for the degenerate gravitating non-rotating spheres corresponding minimum occurs at $y_0^{-2} \approx 0.45$.

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Furodendin, a C_{22} degraded terpene from the sponge *Phyllospongia dendyi*

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Summary. The structure of furodendin (5), a minor secondary metabolite of the sponge *Phyllospongia dendyi*, has been solved by spectral methods. Furodendin is probably derived, biosynthetically, by elimination of a C_3 unit from a C_{25} geranyl-farnesol precursor.

Many species of the sponge genus *Spongia* have been found to contain the biosynthetically intriguing C_{21} difuranoterpenes³ (eg. 1) probably derived from linear sesterterpene tetrone acids³ (eg. 2) found in sponges of the genus *Ircinia*². The sponge genus *Phyllospongia* has yielded a series of C_{26} and C_{27} tetracyclic 'sesterterpenes' (eg. 3) related to scalarin but with additional methyl groups at C24 for the C_{26} compounds and at C19 and C24 for C_{27} representatives^{4,5}.

We have found that *Phyllospongia* species also contain truncated furanoterpenes. Thus *P. foliascens* collected near Cairns on the Great Barrier Reef was found to contain, in addition to C_{27} tetracyclic terpenes, the previously unreported didehydrofuranospongini-1 (4). We have also isolated a novel C_{22} furanoterpene (5) for which we propose the name furodendin, from *P. dendyi*.

Furodendin (5) was isolated as an oil together with the previously reported C_{26} tetracyclic sesterterpenes (3) and (6)⁴ from the dichloromethane extract of the freeze-dried sponge by silica gel chromatography. The formula

$C_{22}H_{30}O_3$ was established by high resolution mass spectrometry. The IR-spectrum (ν_{\max} 1743 cm^{-1}) suggested the presence of an ester or 6-membered ring lactone carbonyl and this was supported by the presence of a singlet at 169.4 in the ^{13}C -NMR-spectrum.

The ^1H -NMR-spectrum of (5) showed resonances typical of a β -substituted furan [δ 7.36 (1H, bs); 7.20 (1H, bs); 6.30 (1H, bs)] and two $\text{CH}_2\text{-CH}=\text{C}(\text{CH}_3)$ -groups [δ 5.20 (1H, bt); 5.12 (1H, bt) and 1.60 (6H, bs)]. The remainder of the spectrum comprised a 2 proton multiplet at δ 1.54, signals due to 6 allylic CH_2 groups between δ 2.6 and 1.9, a 2 proton broad singlet at δ 3.08, a 2 proton broad signal at δ 4.62 and a broad resonance at δ 5.56 (1H).

Irradiation at δ 3.08 sharpened the δ 5.56 and 4.62 signals considerably and irradiation at δ 5.56 gave the δ 4.62 resonance as a dd ($J=2.5, 2.5$ Hz) and the signal at δ 3.08 as a finely coupled multiplet. The ^{13}C -NMR [δ 169.4(s), 142.6(d), 138.8(d), 135.6(s), 135.3(s), 134.2(s), 134.0(d), 125.4(d), 125.3(d), 115.4(s), 111.0(d), 71.0(t), 30.0(t), 28.4(t), 26.5(t), 25.5(t), 25.1(t), 16.0 (q, 2C)] indicated a carbonyl

